

\* Can be exact

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\* Riquetti Equation

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

\* Kirchoff's law

$$M(x,y)dx \neq N(x,y)dy$$

$$\Rightarrow \frac{M_y - N_x}{M} = g(y)$$

$$\therefore \oint = e^{-\int g(y) dy}$$

$$\Rightarrow \frac{M_y - N_x}{N} = f(x)$$

$$\oint = e^{+\int f(x) dx}$$

$$\therefore \left( M(x,y) \oint dx + N(x,y) \oint dy = 0 \right)$$



ex:-  $(1-xy)dx + (xy-x^2)dy = 0$

Solution

$$M = (1-xy)$$

$$N = xy - x^2$$

$$M_y = -x$$

$\neq$

$$N_x = y - 2x$$

$$M_y - N_x = -x - y + 2x = x - y = -(y - x)$$

$$\Rightarrow \int = \frac{M_y - N_x}{N} = \frac{-(y-x)}{x(y-x)} = \frac{-1}{x}$$

$$\Rightarrow \int = e^{\oplus \int f(x)} = e^{\int (-\frac{1}{x}) dx} = \frac{1}{x}$$

$$\therefore (1-xy) \frac{1}{x} dx + (xy-x^2) \frac{1}{x} dy = 0$$

$$dx \left( \frac{1}{x} - y \right) + (y - x) dy = 0$$

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$$\therefore \ln x - yx + (y^2/2) - xy = C$$

$$\ln x - yx + \frac{y^2}{2} = C$$



ex:-  $(4xy + 3y^2 - x).dx + x(x+2y).dy = 0$

Solution

$$M = 4xy + 3y^2 - x \quad N = x^2 + 2yx$$

$$M_y = (4x + 6y) \neq N_x = 2x + 2y$$

$$f = \frac{M_y - N_x}{N} = \frac{2x + 4y}{x(x+2y)} = \frac{2}{x}$$

$$\therefore f = e^{\int f(x) dx} = e^{\int (\frac{2}{x}) dx} = e^{2 \ln x} = x^2$$

بالضرب في  $x^2$   $\rightarrow$   $f = x^2$

$$\therefore (4x^3y + 3y^2x^2 - x^3).dx + (x^4 + 2yx^3).dy = 0$$

بالتكامل

$$\therefore \underbrace{x^4y} + \underbrace{x^3y^2} - \frac{1}{4}x^4 + \underbrace{x^4y} + \underbrace{y^2x^3} = C$$

$$\therefore x^4y + x^3y^2 - \frac{1}{4}x^4 = C$$



$$e^x: - y(x+y+1) dx + x(x^2+3y+2) dy = 0$$

$$M = xy + y^2 + y$$

$$N = x^2 + 3yx + 2x$$

$$M_y = x + 2y + 1 \neq N_x = 2x + 3y + 2$$

$$\int^M = \frac{M_y - N_x}{M} = \frac{-x - y - 1}{-(-x - y - 1)y} = \frac{-1}{y}$$

$$\int^M = e^{\int (-\frac{1}{y}) dy} = y$$

$$\Rightarrow \therefore (y^2x + y^3 + y^2) dx + (x^2y + 3y^2x + 2xy) dy = 0$$

$$\left( \frac{y^2x^2}{2} + y^3x + y^2x \right) + \frac{x^2y^2}{2} + y^3x + xy^2 = C$$

$$\therefore \frac{y^2x^2}{2} + y^3x + y^2x = C$$



# \* Riccati Equation

$$\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x) \rightarrow \textcircled{1}$$

$\Rightarrow$  if  $A(x) = 0 \Rightarrow$  Linear

$\Rightarrow$  if  $B(x) = 0 \Rightarrow$  Can be linear

$\Rightarrow$  if  $F(x)$  is a solution for the <sup>diff</sup> eqn.

$$\Rightarrow y = F(x) + \frac{1}{v(x)}$$

$$\frac{dy}{dx} = \frac{dF}{dx} - \frac{1}{v^2} \frac{dv}{dx} \quad \textcircled{2} \text{ guess!}$$

$$\therefore \frac{dF}{dx} - \frac{1}{v^2} \frac{dv}{dx} = A \left[ F + \frac{1}{v} \right]^2 + B \left[ F + \frac{1}{v} \right] + C$$

$$\frac{dF}{dx} - \frac{1}{v^2} \frac{dv}{dx} = A \left[ F^2 + 2\frac{F}{v} + \frac{1}{v^2} \right] + BF + \frac{B}{v} + C$$

$$\therefore \frac{dF}{dx} - \frac{1}{v^2} \frac{dv}{dx} = AF^2 + 2A\frac{F}{v} + \frac{A}{v^2} + BF + \frac{B}{v} + C$$

$$\therefore F(x) \text{ hai hai} \therefore \frac{dF}{dx} = AF^2 + BF + C$$

$$\therefore -\frac{1}{v^2} \frac{dv}{dx} = 2A\frac{F}{v} + \frac{A}{v^2} + \frac{B}{v}$$



$$\therefore \frac{dv}{dx} = 2AfV + A + Bv$$

$$\therefore \frac{dv}{dx} = V(2Af+B) + A$$

$$\therefore \boxed{\frac{dv}{dx} + V(2Af+B) = -A}$$

ex:-  $\frac{dy}{dx} = -8xy^2 + 4x(4x+1)y - (8x^3 + 4x^2 - 1)$

→ if the function  $f(x) = x$  is a solution find  $y$ ?

{ Solution }

$$A = -8x$$

$$B = 16x^2 + 4x$$

$$C = -(8x^3 + 4x^2 - 1)$$

$$\Rightarrow \frac{dv}{dx} + V(2Af+B) = -A$$

$$\frac{dv}{dx} + V(-16x^2 + 16x^2 + 4x) = +8x$$

$$\frac{dv}{dx} + 4xV = 8x$$

→ Linear

$$\Rightarrow I = e^{\int 4x dx} = e^{2x^2}$$

$$V = \frac{1}{e^{2x^2}} \left[ \int e^{2x^2} * \frac{8x dx}{2(4x)} + C \right]$$

$$V = \frac{1}{e^{2x^2}} \left[ 2e^{2x^2} + C \right] \quad e^{\int 4x dx}$$

$$\therefore y = f + \frac{1}{V_1} = x + \boxed{\frac{1}{V_1}}$$



ex:  $\frac{dy}{dx} = (1+x+x^2) - (1+2x)y + y^2$

if  $y = x$  ??? find  $y$

$A = 1$        $B = -(1+2x)$        $C = (1+x+x^2)$

$\Rightarrow \frac{dv}{dx} + v(2Ax+B) = -A$

$\frac{dv}{dx} + v(2x + (-1-2x)) = -1$

$\frac{dv}{dx} - v = -1 \quad \Rightarrow I = e^{-x}$

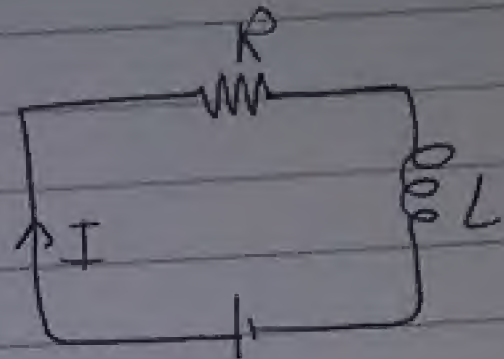
$v = \frac{1}{e^{-x}} \left[ \int -e^{-x} dx + c \right]$



## Kirchoff's law

$$\mathcal{E} = RI + L \frac{dI}{dt}$$

find  $I$  ??



$$\frac{dI}{dt} + \frac{R}{L} I = \frac{\mathcal{E}}{L} \rightarrow \text{linear}$$

$$I = e^{\int \left(\frac{R}{L}\right) dt} = e^{\frac{R}{L}t}$$

$$I(t) = \frac{1}{e^{\frac{R}{L}t}} \left[ \int \frac{\mathcal{E}}{L} e^{\frac{R}{L}t} dt + C \right]$$

$$I = \frac{1}{e^{\frac{R}{L}t}} \left[ \frac{\mathcal{E}}{L} \frac{L}{R} e^{\frac{R}{L}t} + C \right] = \frac{1}{e^{\frac{R}{L}t}} \left[ \frac{\mathcal{E}}{R} e^{\frac{R}{L}t} + C \right]$$

$$I = \frac{\mathcal{E}}{R} + C e^{-\frac{R}{L}t}$$

at  $t=0 \rightarrow I(0)=0$

$$\therefore 0 = \frac{\mathcal{E}}{R} + C(1) \rightarrow C = -\frac{\mathcal{E}}{R}$$

$$I = \frac{\mathcal{E}}{R} (1 - e^{-\frac{R}{L}t})$$

ففي المبدئ  $t=0$  لا يمكن يداوغي  $R$  و  $L$   
ويقل افع التيار و توابع  
الكل  $\leftarrow$  شغل